MATH 54 – MOCK FINAL EXAM

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Name: ____

Instructions: This is a mock final, designed to give you an idea of what the actual final will look like!

1		10
2		15
3		15
4		30
5		15
6		15
Total		100

Date: Friday, August 10th, 2012.

1. *(10 points, 2 points each)*

Label the following statements as **T** or **F**. **Write your answers in the box below!**

NOTE: In this question, you do **NOT** have to show your work! Don't spend *too* much time on each question!

(a) If Q has orthogonal columns, then Q is an orthogonal matrix

(b) If $\hat{\mathbf{x}}$ is the orthogonal projection of $\mathbf{x}$ on W, then $\mathbf{x}-\hat{\mathbf{x}}$ is always orthogonal to $\hat{\mathbf{x}}$.

(c) The least-squares solution $\widetilde{\mathbf{x}}$ of $A\mathbf{x} = \mathbf{b}$ has the property that $\|A\mathbf{x} - \mathbf{b}\| \leq \|A\widetilde{\mathbf{x}} - \mathbf{b}\|$ for every $\mathbf{x}$

(d) If a set $\mathcal{B}$ is orthogonal, then $\mathcal{B}$ is linearly independent

(e) $\begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = ac$ defines a dot/inner product on $\mathbb{R}^2$.

(a)	
(b)	
(c)	
(d)	
(e)	

2. *(15 points)* Use the Gram-Schmidt process to find an orthonormal basis for W, where:

$$W = Span\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} \right\}$$

3. *(15 points)* Find the least-squares solution and least-squares error to the following (inconsistent) system of equations $A\mathbf{x} = \mathbf{b}$, where:

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

4. *(30 points)* Solve the following heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} & 0 < x < 1, \quad t > 0 \\ u(0,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = x & 0 < x < 1 \end{cases}$$

Note: You may not use **ANY** for the formulas given in the book! You have to do it from scratch, including the 3 cases.

Note: The following formula might be useful:

$$\int_{-1}^{1} \cos^2(\pi m x) = \int_{-1}^{1} \sin^2(\pi m x) = 1$$

(Scratch work)

5. *(15 points)*

(a) *(10 points)* Find the Fourier cosine series of $f(x) = x^2$ on $(0, \pi)$

That is, find A_m such that:

$$x^2 \text{``}=\text{''} \sum_{m=0}^{\infty} A_m \cos(mx) \quad \text{on} (0, \pi)$$

Hint: The following formula might be useful:

$$\int_{-\pi}^{\pi} \cos^2(mx) = \int_{-\pi}^{\pi} \sin^2(mx) = \pi$$

(b) *(5 points)* Draw the graph of the function to which the above Fourier series $\mathcal{F}$ converges to on $(-3\pi, 3\pi)$

6. *(15 points)*

Prove the parallelogram identity:

$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

Note: Do it in general, not just for $\mathbb{R}^n$